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Question Paper Code : 23769

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2018.

Fourth Semester

Electronics and Communication Engineering

MA 2261 — PROBABILITY AND RANDOM PROCESSES

(Common to Biomedical Engineering)

(Regulations 2008)

Time : Three hours   Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. If $P(A)=0.4$, $P(B)=0.7$, $P(A \cap B)=0.3$, find $P(A' \cap B')$.

2. The random variable X has the following distribution :

X:	-2	-1	0	1
$P(X)$:	0.4	K	0.2	0.3

Find K and the mean value of X.

3. Find the marginal distributions of X and Y from the bivarite distribution given by .

X \ Y	1	2
1	0.1	0.2
2	0.3	0.4

4. The two regression equations are $x = 19.13 - 0.87y$ and $y = 11.64 - 0.50x$. Find the correlation coefficient.

5. State any two properties of Poisson process.

6. Define recurrent and transient state.

7. Define auto-correlation.

8. Find the mean and variance of the stationary process $(X(t))$, whose ACF is given by $R(\tau) = \dfrac{25\tau^2 + 36}{6.25\tau^2 + 24}$.

9. Define white noise Processes.

10. Define linear time invarient system.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find the mean, variance and moment generating function of a Poisson random variable.

(ii) A random variable X has the density function

$$f(x) = \begin{cases} x/4, & -2 < x < 2 \\ 0, & \text{otherwise} \end{cases}$$

Obtain

(1) $P(X < 1)$

(2) $P(|X| > 1)$

(3) $P(2X + 3 > 5)$. (8)

Or

(b) (i) The mileage in thousands of miles which car owners get with a certain kind of tyre is a random variable having a probability density function

$$f(x) = \begin{cases} 1/20\, e^{-x/20}, & \text{for } x > 0 \\ 0, & \text{otherwise} \end{cases}$$

Find the probability that one of these tyres will last for

(1) atmost 10,000 miles

(2) from 16,000 to 24,000 miles

(3) at least 30,000 miles. (8)

(ii) Find the mean, variance and the moment generating function of a uniformly distributed random variable. (8)

12. (a) If X and Y are two random variables having joint density function

$$f(x,y) = \begin{cases} (6 - x - y)/8, & 0 < x < 2,\ 2 < y < 4 \\ 0, & \text{otherwise} \end{cases}$$

Find

(i) $P(X < 1 \cap Y < 3)$

(ii) $P(X + Y < 3)$

(iii) $P(X < 1 / Y < 3)$

(iv) Covariance between X and Y. (16)

Or

(b) Find the coefficient of correlation and obtain the lines of regression from the data given below. (16)

x:	62	64	65	69	70	71	72	74
y:	126	125	139	145	165	152	180	208

13. (a) (i) Show that the random process $X(t) = A\cos(wt+\theta)$ is a WSS process if A and W are constants and θ is uniformly distributed in $(0, 2\pi)$. (8)

(ii) The t.p.m of a Markov chain with three states 0, 1, 2 is

$$P = \begin{bmatrix} \frac{3}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \\ 0 & \frac{3}{4} & \frac{1}{4} \end{bmatrix}$$

and the initial state distribution of the chain is $P[X_0 = i] = \frac{1}{3}$, $i = 0, 1, 2$.

Find

(1) $P[X_2 = 2]$

(2) $P[X_3 = 1, X_2 = 2, X_1 = 1, X_0 = 2]$. (8)

Or

(b) (i) A man either drives a car or catches a train to go to office each day. He never goes 2 days in row by train, but if he drives one day, then the next day he is just as likely as to drive again as he is just as likely as to drive again as he is travel by train. Now suppose that on the first day of the work the tossed a die and drove to work if and only if a 6 appeared. Find the

(1) Probability that he takes a train on the third day

(2) Probability that he drives to work in the long run? (8)

(ii) Prove that the sum of two independent Poisson process is also a Poisson process. (8)

14. (a) If $X(t)$ and $Y(t)$ are zero mean and stochastically independent random processes having auto correlation functions $R_{XX}(\tau) = e^{-|\tau|}$ and $R_{YY}(\tau) = 2\cos\pi\tau$ respectively. Find the ACF of

(i) $w(t) = X(t) + Y(t)$

(ii) $Z(t) = X(t) - Y(t)$. (16)

Or

(b) A random process $(X(t))$ is given by $X(t) = A\cos pt + B\sin pt$, where A and B are independent RVs such that $E(A) = E(B) = 0$ and $E(A^2) = E(B^2) = \sigma^2$. Find the power spectral density of the process. (16)

15. (a) (i) The input to the RC filter is a white noise process with ACF $R_{XX}(\tau) = \frac{N_0}{2}\delta(\tau)$. If the frequency response $H(w) = \frac{1}{1 + jwRC}$. Find the ACF and the mean square value of the output process $Y(t)$. (8)

(ii) A linear time invariant system has an impulse response $h(t) = e^{-at}u(t)$. Find the output ACF corresponding to an input $X(t)$. (8)

Or

(b) (i) If the input $X(t)$ and its output $Y(t)$ are related by $Y(t) = \int_{-\infty}^{\infty} h(u)\, X(t-u)\, du$ then show that the system is a linear time-invariant system. (8)

(ii) A linear system is described by the impulse response $h(t) = \frac{1}{RC} e^{\frac{-t}{RC}} u(t)$. Assume an input process whose auto correlation function is $A\delta(\tau)$. Point out the mean and the autocorrelation function of the output function. (8)